MERIA scenario “Area enlargement”

Target knowledge	Whenever all side lengths of a polygon are enlarged by a certain factor k, the area of the polygon is enlarged by the factor k^2.
Broader goals	Autonomous algebraic and geometric reasoning, formulation of general statements and proofs based on formulas of circumferences and areas of different shapes, possibly including the sine function as well as additivity of area when cutting the polygon in parts. The notion of similar polygons. If students are used to work with ICT: to generate hypotheses in a graphical environment and use it as an outset of a proof.
Prerequisite mathematical knowledge	Students need to have some knowledge of how to compute area of polygons, including triangles and squares. Also needed: notion of similarity, magnifying polygons by a scale factor.
Grade	Grade 10, students aged 15-16 years
Time	90 minutes, two lessons
Required material	Pen, paper, grid paper, ruler, a mathematics tool which allows to draw and measure polygons (e.g. Geogebra), a device which allows to manipulate pictures (smartphone, pc, or tablet). The use of technology is strictly speaking not needed but greatly enhances the familiarity of the environment for most students.

Observations from implementation

Context of observations (grade, institution, country, etc.):

Problem:

Look at these two pictures. If you open them on your smartphone or computer, you can easily drag the pictures in order to enlarge them. But what happens with the areas of the pyramid and of the black building when we enlarge the picture?

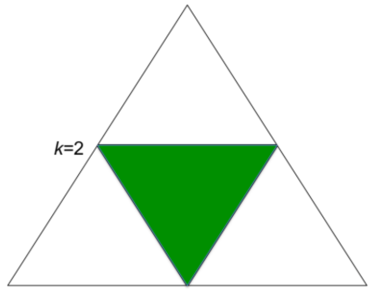
Phase	Teacher's actions incl. instructions	Students' actions and reactions	Observations from implementation
Devolution (didactical) 2 minutes	The teacher starts by asking: What do I need to know in order to find the area of a triangle? Or the area of any other polygon? There is more than one answer and you are welcome to provide several answers. Write down the answer on a piece of paper. You have 2 minutes to do this.	Students accept the task, and possibly ask clarifying questions to make sure they understand their task.	
Action and formulation (adidactical) 2 minutes	The teacher walks around the room and identifies what different ideas the students recall and write down.	The students write down formulas such as $A_{square} = l \cdot h,$ $A_{triangle} = \frac{h \cdot b}{2},$ $A = \frac{a \cdot b \cdot \sin(C)}{2}.$ They may also note that the area of any polygon may be calculated using triangulation. Other methods: Counting squares on a grid paper; Computer based methods.	
Validation (didactical) 5 minutes	The teacher chooses students to present their writings at the board in order to get all strategies represented. The teacher asks the class to pose questions or comments to the presentations.	The students listen to those presenting, and ask for elaboration, comment or discuss the suggestions at the board.	
Institutionalisation (didactical) 2 minutes	The teacher sums up the number of ways to find areas of polygons.	Students listen.	

Devolution (didactical) 2 minutes	The students are divided into groups of 3, but should start working individually. They have 15 minutes to prepare their own answer to the problem stated above. Ask if students understand it. The students are provided with (or asked to bring) plane paper, grid paper, scissor, ruler, calculator and computer with relevant ICT.	Students listen and pose clarifying questions if needed. They pick up materials needed, if they want to use paper, ruler etc.	
Action (adidactical) 15 minutes	The teacher circulates the classroom to note what strategies students choose. *The teacher does not interact except to clarify the problem*.	Students start to try some of the strategies in their group. For possible strategies see below.	
Formulation (adidactical) 10 minutes	The teacher asks the groups to agree upon one answer to the problem by presenting and discussing their personal ideas. The teacher surveys the group work so (s)he can organise the presentations.	Students give a short presentation of their work and the group refines the presentation of the chosen strategy.	
Validation (didactical) 20 minutes	The teacher calls the groups to present one by one, starting with the most practical and vague formulations, and ending up with the most general arguments. The class is encouraged to pose elaborating questions together with the teacher during other groups' presentations.	Students give their best possible presentations, listen and pose elaborating questions if other presentations are unclear to them.	
Devolution (didactical) 2 minutes	The teacher asks the groups to explain relations and differences among the answers presented. Which is "most useful" and why?	Students accept the task.	

Action/formulation (adidactical) 15 minutes	(S)he observes the arguments formulated in groups.	Student might build arguments on examples, calculations or algebraic manipulations.	
Validation (didactical) 10 minutes	The teacher uses the knowledge about the work of the individual groups to sequence and select different presentations of answers, so all strategies are represented.	Groups present their answers by using the board. Other groups pose clarifying questions or add comments when relevant.	
Institutionalisation (didactical) 5 minutes Total: 90 minutes	The teacher sums up by emphasizing the different strategies. (S)he formulates how the strategies are related and support each other, though some strategies are preferred in certain cases (e.g. new examples). The teacher formulates the target knowledge in its general form, pointing out how it appears in the different solutions proposed by students.	Students listen and some might take notes.	

Possible ways for students to realize target knowledge	○ Drawing the polygons on grid paper, counting the number of covered squares in the original and enlarged figures (without explicit use of the concept of scale factor of lengths).
	○ Drawing on any piece of paper, measuring the baseline, b, and the height, h, with a ruler in order to calculate the area using $A = \frac{h \cdot b}{2}$ (again without scale factor of side lengths).
	○ Experimenting with different scale factors of lengths (2, 3, 0.5 etc.), arriving at hypotheses such as employing a factor 2 leads to areas increased by 4, etc.
	    ○ Can be realized algebraically from examples (choose dimension of a triangle).
	    ○ The above strategy can be carried out using ICT.
	    ○ It can be realized with grid paper. Draw different shapes, enlarge them and count the number of squares covered.
	    ○ It can be realized by drawing triangles on paper, use the ruler to measure the side lengths and calculate the areas.
	    ○ The shapes can be drawn using ICT, e.g. Geogebra. Side lengths and areas may be measured using instrumented techniques, depending on the tool.

- Draw the triangles and cut them out of the paper. If the enlargement is an integer, k, then the small triangle can be fitted k^2 times in the enlarged one.



- Using ICT: draw the polygon, drag it until it is enlarged by a certain amount and ask the program to calculate the areas. For instance, students can use geometric sketchpad for enlarging pictures and observe what happens to the area.
- Based on experiments as described above the following symbolic reasoning can be developed:
  - If we increase the side length of a right-angled triangle (having height h and base line b) by a factor k, then the new height will be $k \cdot h$ and the base line $k \cdot b$. This means that the area is increased by a factor k^2, since $A_2 = \frac{1}{2} \cdot kh \cdot kb = k^2 \cdot A_1$, where A_1 is the area of the initial triangle.
  - For the general triangle with height h and baseline b one needs to argue why the height increases to $k \cdot h$. This could be done by looking at the two right triangles which make up the general triangle.
  - If we increase the side length of an arbitrary triangle (having two sides a, b and intermediate angle C) by a factor k, then the initial area can be calculated as $A_1 = \frac{1}{2} \cdot a \cdot b \cdot \sin(C)$. The increased area is then $A_2 = \frac{1}{2} \cdot ka \cdot kb \cdot \sin(C) = k^2 \cdot A_1$
  - If we look at equal sided triangles then they can use the formula $A = \frac{\sqrt{3}}{4} s^2$, so if s increases by k, the area increases by a factor k^2
  - For the right hand side picture (and other polygons) divide the polygon into triangles and calculate the sum of areas, using some of the above mentioned methods for triangles.
  - They can use Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$. This requires students to be able to conduct algebraic manipulations of expressions containing roots and powers.