MERIA Scenario “ab-ba”

The Distributive Law

Target knowledge	Using the distributive law $n(a+b) = n\,a + n\,b$.
Broader goals	Inquiry skills. Problem solving skills.
Prerequisite mathematical knowledge	Basic arithmetic.
Grade	13 year olds
Time	20-25 minutes
Required material	Pen and paper, possibly calculator
Observations from implementation Context of observations (grade, institution, country, etc.):	
Problem: Look at two-digit numbers, for example 83. For each number look at the difference between the number and its reverse (for 83 this is 38). What do you get if you subtract the smaller one from the bigger one (83-38)? Try again with new numbers. What pattern do you see? Can you explain this?	

Phase	Teacher’s actions incl. instructions	Students’ actions and reactions	Observations from implementation
Devolution (didactical) 3 minutes	The teacher states the problem as above, including the example of 83 to make sure the students understand what is meant by “reverse” and what calculation to make.	Students listen, calculate 83-38=... They try with new numbers and try to understand the problem.	
Action (adidactical) 10 – 15 minutes	Teacher walks round the classroom and registers the students’ strategies.	Students try to solve the problem.	

Formulation (Adidactical or didactical if the teacher feels support is needed) 3 minutes	The teacher invites for each significantly different strategy one group to formulate their solution on the blackboard. If students find a solution quickly, the teacher could suggest they try the same problem for more-digit numbers.	Chosen students present their solution. The other students listen, compare their own work to the presented solution and ask questions. There are two options: (1) The groups just found that the difference is divisible by 9. (2) The students found a justification in the form of approaches (A), (B), (C), (D) or (E).	
Validation (didactical) 7 minutes	In case (1) the teacher could lead a classroom discussion on how to know for sure the hypothesis is true for all 2-digit numbers. The outcome could be approach (A), (B), (C), (D) or (E) below which is discussed and then forms a validation of the students' hypothesis.		
Institutionalisation (didactical or adidactical) 5 minutes (or more)	The teacher explains how the step $9a - 9b = 9(a - b)$ or $9 \cdot 8 - 9 \cdot 3 = 9(8 - 3)$ is an instance of a more abstract mathematical law $n(a + b) = n\,a + n\,b$. The teacher can show more instances of this law.		

Possible ways for students to realize target knowledge	• Approach (A): algebraic approach Say the number equals $10a + b$ for $a = 1,2, \dots, 9$ and $b = 0,1,2, \dots, 9$. Then the reverse is $10b + a$. The difference between these numbers is plus or minus $10a + b - (10b + a) = 9a - 9b = 9(a - b)$. • Approach (B): implicit algebraic (by numerical example) Say the number equals $83 = 10 \cdot 8 + 3$. Then the reverse is $38 = 10 \cdot 3 + 8$. The difference between these numbers is plus or minus $$83 - 38 = 10 \cdot 8 + 3 - (10 \cdot 3 + 8) = 9 \cdot 8 - 9 \cdot 3 = 9(8 - 3) = 9 \cdot 5.$$ • Approach (C): First notice that the claim is true for numbers in the multiplication table of 9, for it contains all its reverses: 09 and 90, 18 and 81, 27 and 72 etc. Then note that starting with one of those numbers adding 1 to it means adding 10 to the reverse, adding 2 to the first means adding 20 to the reverse, etc. For the difference this means either adding plus or minus 10-1 or 20-2 or 30-3 etc. which is again the multiplication table of 9. As an example, consider 39. This is 36 + 3, where 36 is in the table of 9. So, 36+3-(63+30) is divisible by 9 because 36, 63 and 3-30 are. • Approach (D): This approach uses the criterium that a number is divisible by 9, if and only if the sum of its digits is divisible by 9. We denote a number 35 by [3.5] to be able to keep track of its decimal digits. Suppose you chose number $[a.b]$ with reverse $[b.a]$. Assume $a > b$, then the difference is $[a.b] - [b.a] = [(a-1) - b.(10+b) - a]$. The sum of the digits is $a - 1 - b + (10 + b - a) = 9$ and the result follows. Example: $[5.3] - [3.5] = [4 - 3.13 - 5]$ and $4 - 3 + 13 - 5 = 9$. • Approach (E): The claim is true for 1. Because 1-10=-9. Then we prove for the other numbers inductively. Suppose it is true for number n. Adding 1, then $$n + 1 - (n+1)_{reverse} = n + 1 - (n_{reverse} + 10) \text{ or } n + 1 - (n+1)_{reverse} = n + 1 - (n_{reverse} - 89).$$ Both -9 and +90 are multiples of 9. So, the result is a multiple of 9.